(高中知识复习)指数函数的图像

$y=a^x$ $(a>0$ 且 $a\neq 1)$

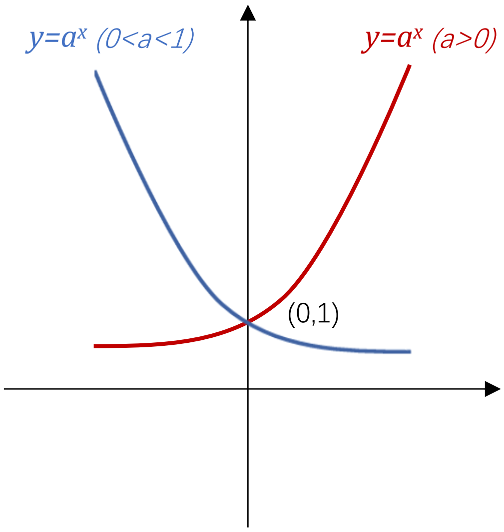
性质: (1) Domain: $x\in R$,

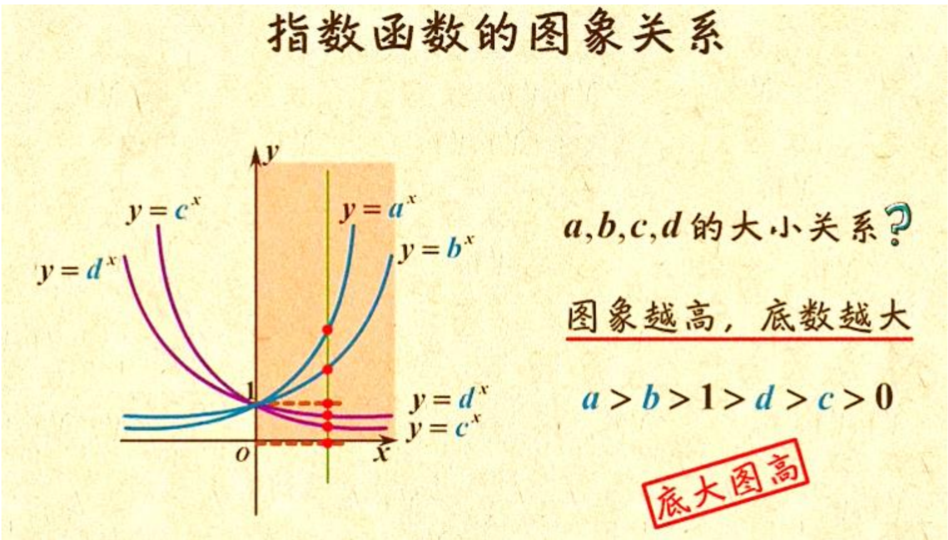
Range: $y\in(0,+\infty)$

(2) 过定点(0,1)

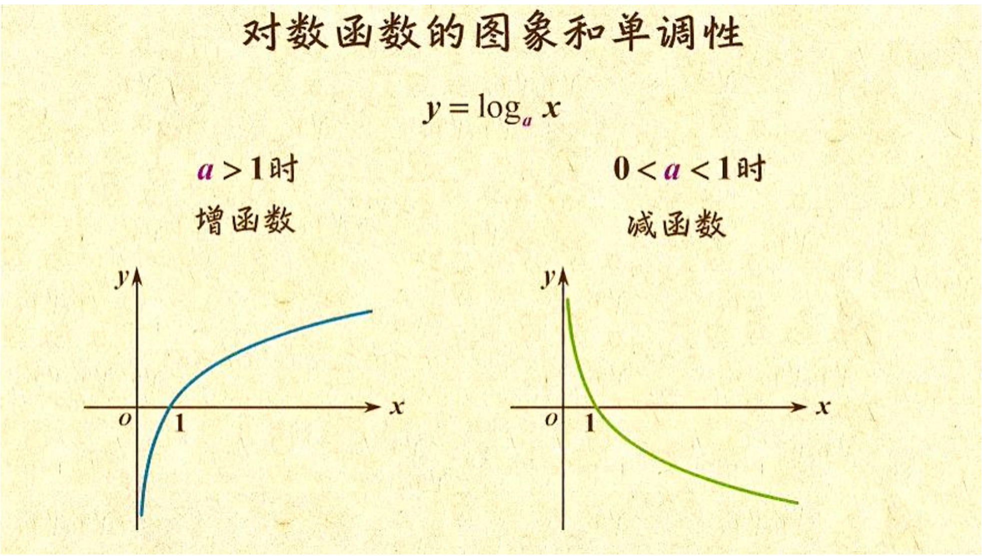
(3) a>1: 增函数

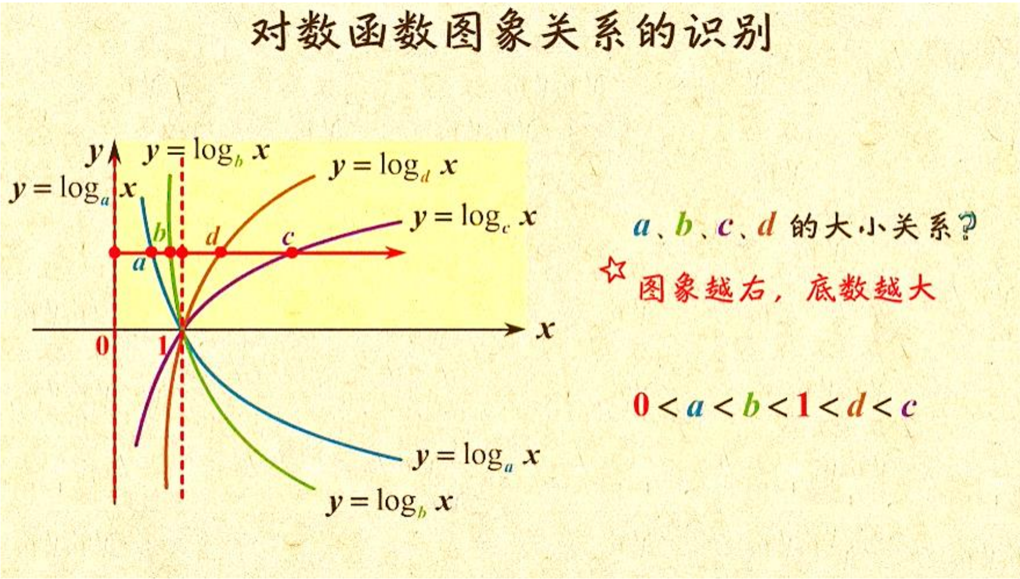
0<a<1: 减函数





(高中知识复习)对数函数的图像





Exponential/Logarithmic Function 指数/对数函数

The Differentiation of Exponential Function 指数函数微分

$$f(x) = a^x$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{a^{x+\Delta x} - a^x}{\Delta x} = \lim_{\Delta x \to 0} \frac{a^x \cdot a^{\Delta x} - a^x}{\Delta x} = \lim_{\Delta x \to 0} \frac{a^x(a^{\Delta x} - a^0)}{\Delta x} = a^x \lim_{\Delta x \to 0} \frac{(a^{\Delta x} - a^0)}{\Delta x}$$

Please notice that a^x does not vary with variation in the value of Δx.

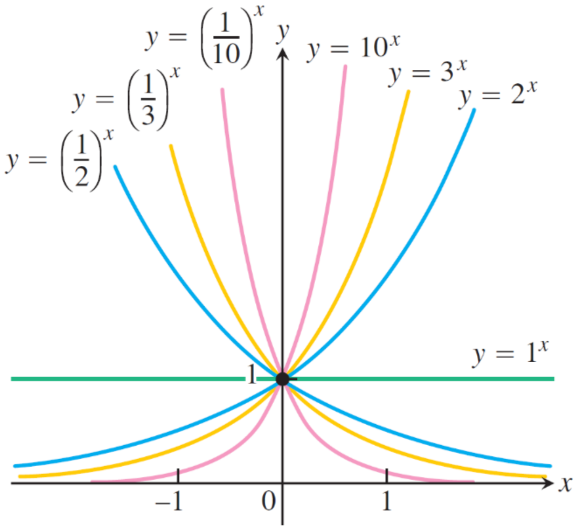
So we have:

$$\frac{d}{dx}(a^x) = f'(a^x) = a^x \lim_{\Delta x \to 0} \frac{(a^{\Delta x} - a^0)}{\Delta x} = a^x \cdot L$$

Definition of number *e* 常数 *e* 的定义:

$\lim_{\Delta x \to 0} \frac{(a^{\Delta x} - a^0)}{\Delta x}$ is the slop of a^x at point $(0,1)$.

$\frac{d}{dx}(a^x) = a^x \cdot L$. This means the slop of $y = a^x$ is a^x times the slop a^x at point $(1,0)$.



Let's evaluate $L = \lim_{\Delta x \to 0} \frac{(a^{\Delta x} - a^0)}{\Delta x} = f'(0)$:

a	0.5	1.0	1.5	2.0	2.5	3.0	3.5
L	-0.693	0.000	0.405	0.693	0.916	1.099	1.253

From the table, it seems that there exists a value of a between 2.5 and 3.0 such that the slop of $f(x) = a^x$ at $(0,1) = 1$.

a	2.5	2.6	2.7	2.8	2.9
L	0.916	0.956	0.993	1.030	1.065

We call this **special value *e***.

$\boldsymbol{e} \approx 2.71828$

Differentiation of Exponential Function 指数函数微分

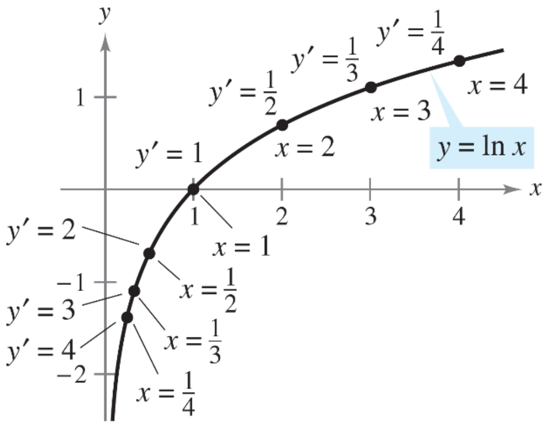
$$\frac{d}{dx}(e^x) = e^x$$

The Differentiation of Logarithmic Function 对数函数微分

Let $y = ln\,x$

$$e^y = x \Rightarrow \frac{d}{dx}(e^y) = e^y\frac{dy}{dx} = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$$



$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

For same reason $y = ln(-x)$

We can also obtain

$$\frac{d}{dx}[\ln(-x)] = \frac{1}{x}$$

In general

$$\frac{d}{dx}\ln|x| = \frac{1}{x}$$

Example Find the derivative of the exponential functions:

(a) a^x. Let $y = a^x \Rightarrow \ln y = \ln(a^x) = x\ln a$

$$\Rightarrow \frac{d}{dx}(\ln y) = \frac{d}{dx}(x\ln a) \Rightarrow \frac{1}{y}\frac{dy}{dx} = \ln a \Rightarrow \frac{dy}{dx} = y\ln a \qquad \frac{d}{dx}(a^x) = a^x\ln a$$

(b) $\log_a x \qquad \log_a x = \frac{\ln x}{\ln a} \Rightarrow \frac{d}{dx}(\log_a x) = \frac{d}{dx}\left(\frac{\ln x}{\ln a}\right) = \frac{1}{x\ln a}$

(c) $e^{2x} \qquad \frac{d}{dx}(e^{2x}) = e^{2x}\cdot 2$ by chain rule

Example Differentiating Exponential Functions

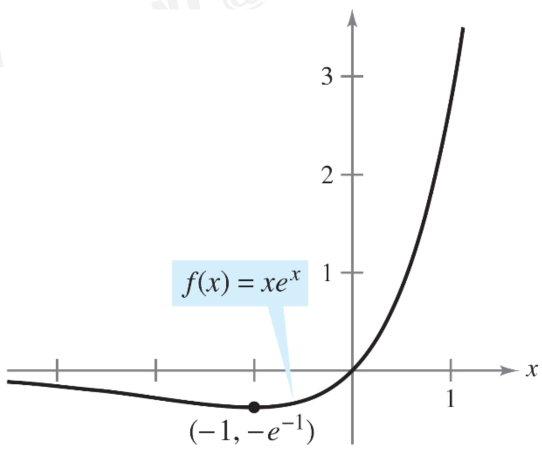
(a) $\frac{d}{dx}(e^{2x-1}) = e^u\frac{du}{dx} = 2\,\mathrm{e}^{2x-1} \qquad u = 2x-1$

(b) $\frac{d}{dx}(e^{-\frac{3}{x}}) = e^u\frac{du}{dx} = (\frac{3}{x^2})e^{-\frac{3}{x}} \qquad u = \frac{-3}{x}$

Example Find the relative extrema of $f(x) = xe^x$

$f'(x) = xe^x + e^x(1)$

$= e^x(x+1) = 0 \xRightarrow{e^x \neq 0} x = -1$



Example Expanding Logarithmic Expressions

(a) $\ln\frac{10}{9} = \ln 10 - \ln 9$

(b) $\ln\sqrt{3x+2} = \ln(3x+2)^{\frac{1}{2}} = \frac{1}{2}\ln(3x+2)$

(c) $\ln\frac{(x^2+3)^2}{x\sqrt[3]{x^2+1}} = 2\ln(x^2+3) - \left[\ln x + \frac{1}{3}\ln(x^2+1)\right]$

$= 2\ln(x^2+3) - \ln x - \frac{1}{3}\ln(x^2+1)$

Example Differentiation of Logarithmic Functions

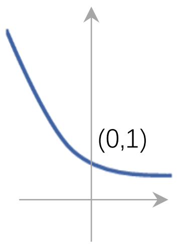
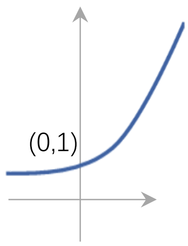
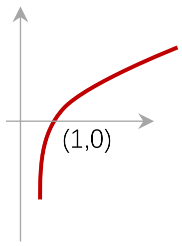
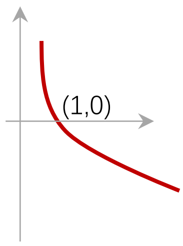
(a) $\frac{d}{dx}[\ln(2x)] = \frac{u'}{u} = \frac{2}{2x} = \frac{1}{x} \qquad u = 2x$

(b) $\frac{d}{dx}[\ln(x^2+1] = \frac{u'}{u} = \frac{2x}{x^2+1} \qquad u = \mathrm{x}^2+1$

(c) $\frac{d}{dx}[x\ln x] = \ln x + x\frac{1}{x} = \ln x + 1$ product rule

(d) $\frac{d}{dx}[(\ln x)^3] = 3(\ln x)^2\left(\frac{1}{x}\right) = \frac{3}{x}(\ln x)^2$ chain rule

Properties 指数对数函数的性质

Function	Exponential $y = a^x$		Logarithmic $y = \log_a x$	
Base	$0 < a < 1$	$a > 1$	$a > 1$	$0 < a < 1$
Domain	$(-\infty, \infty)$	$(-\infty, \infty)$	$(0, \infty)$	$(0, \infty)$
Range	$(0, \infty)$	$(0, \infty)$	$(-\infty, \infty)$	$(-\infty, \infty)$
I/D	Decreasing	Increasing	Increasing	Decreasing
Concavity	C-upward	C-upward	C-downward	C-upward
Graph				
Identity	$a^{log_a n} = n$		$\log_a(a^n) = n$	
Operation	$(ab)^n = a^n b^n$ $(a^m)^n = a^{mn}$ $a^m \times a^n = a^{m+n}$ $a^m \div a^n = a^{m-n}$		$\log_a m + \log_a n = \log_a(mn)$ $\log_a m - \log_a n = \log_a(m \div n)$ $n\log_a m = \log_a(m^n)$ $\log_{a^m} b^n = \frac{n}{m}\log_a b$ $\log_a b = \frac{\log_c b}{\log_c a}$ $\log_a b \times \log_b a = 1$	
Derivative	$(e^x)' = e^x$ $(a^x)' = a^x \ln a$		$(\ln\lvert x\rvert)' = \frac{1}{x}$ $\left(\log_a x\right)' = \frac{1}{x\ln a}$	

姓名: ________________ Name:________________

5 Steps to a 5: AP Calculus BC 2019, William Ma, (p97 6.8)

Part A – The use of a calculator is not allowed.

Find the derivative of each of the following functions.

7. $y = 10\cot(2x-1)$

8. $y = 3x\sec(3x)$

Part B – Calculators are not allowed.

16. $f(x) = x^5 + 3x - 8$, find $(f^{-1})'(-8)$.

17. Write an equation of the tangent to the curve $y = \ln x$ at $x = e$.

26. Find $\lim\limits_{h\to 0} \dfrac{\sin\left(\frac{\pi}{2}+h\right) - \sin\left(\frac{\pi}{2}\right)}{h}$.

28. Find $\lim\limits_{x\to\infty} \dfrac{x-25}{10+x-2x^2}$.

Exercise 1: Known $2^a=5^b=m$, and $\frac{1}{a}+\frac{1}{b}=2$, Find m.

$a=\log_2 m$, $1/a=\log_m 2$.
$b=\log_5 m$, $1/b=\log_m 5$.

$1/a+1/b=2$, $\log_m 2+\log_m 5=2$, $\log_m(2x5)=2$, $m=\sqrt{10}$

Exercise 2: 已知 $\log_{14}7=a$, $\log_{14}5=b$，用 a,b 表示 $\log_{35}28$.

此类题往往分成两步：1. 用换底公式换为已知条件的底。2. 将 log 的真数拆为底数乘以或除以一个数，一直拆下去。

$$log_{35}28=\frac{log_{14}28}{log_{14}35}=\frac{1+log_{14}2}{log_{14}(5\times 7)}=\frac{1+log_{14}(14\div 7)}{log_{14}5+log_{14}7}=\frac{1+1-a}{a+b}=\frac{2-a}{a+b}$$

Exercise 3: (p358-24) Use a graphing utility to graph the function. Use the graph to determine any asymptotes of the function.

(a) $f(x)=\frac{8}{1+e^{-0.5x}}$

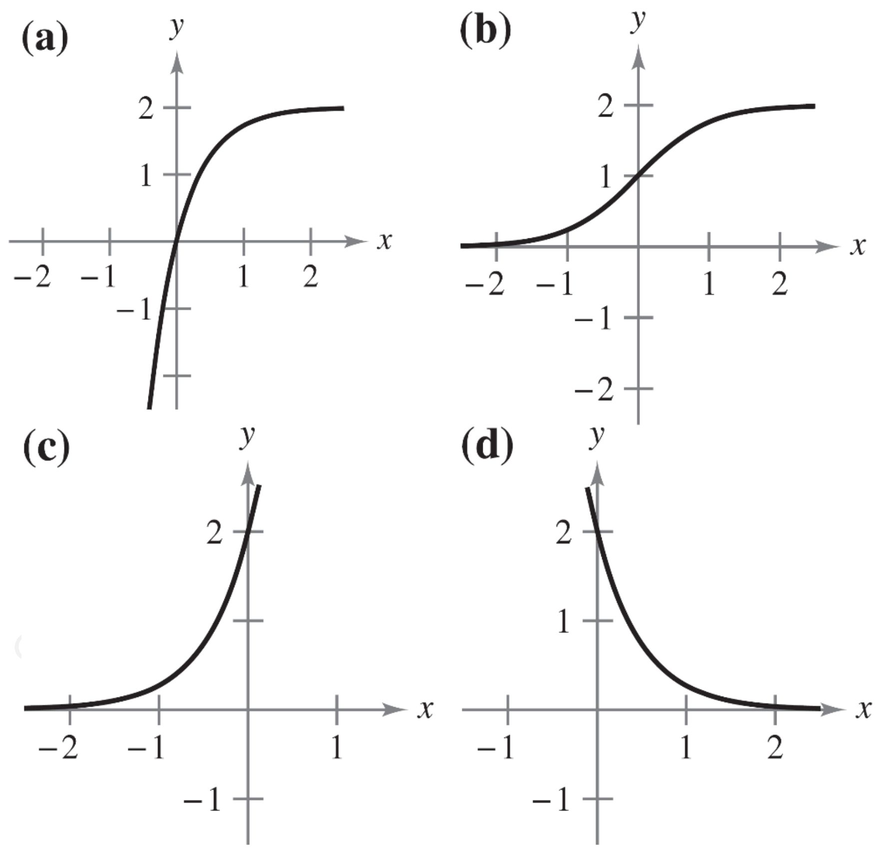
$x\to-\infty, (e^{-0.5x})\to\infty, f(x)\to 0^+$
$x\to\infty, (e^{-0.5x})\to 0, f(x)\to 8^-$

(b) $g(x)=\frac{8}{1+e^{-0.5/x}}$

$$x\to-\infty, \left(\frac{-0.5}{x}\right)\to 0^+, \left(e^{-\frac{0.5}{x}}\right)\to 1^+, \left(1+e^{-\frac{0.5}{x}}\right)\to 2^+\to g(x)\to 4^-$$

$$x\to\infty, \left(\frac{-0.5}{x}\right)\to 0^-, \left(e^{-\frac{0.5}{x}}\right)\to 1^-, \left(1+e^{-\frac{0.5}{x}}\right)\to 2^-\to g(x)\to 4^+$$

Exercise 4: (p358-25), match the equation (labeled a, b, c and d) with the correct graph. Assume that a and C are positive real numbers.



25 $y=Ce^{ax}$
$x\to-\infty, e^{ax}\to 0$
$x\to\infty, e^{ax}\to\infty$
$x=0, y=Ce^{ax}=C=2$
Graph (c)

26 $y=Ce^{-ax}$
$x\to-\infty, e^{ax}\to\infty$
$x\to\infty, e^{ax}\to 0$
$x=0, y=Ce^{-ax}=C=2$
Graph (d)

27 $y=C(1-e^{-ax})$
$x\to-\infty, e^{-ax}\to\infty, y\to-\infty$
$x\to\infty, e^{-ax}\to 0, y\to C=2$
$x=0, y=C(1-e^{-ax})=0$
Graph (a)

28 $y=\frac{C}{1+e^{-ax}}$
$x\to-\infty, e^{-ax}\to\infty, y\to 0$
$x\to\infty, e^{-ax}\to 0, y\to C=2$
Graph (b)

习题答案

Exercise 5: (p359-45), find the derivative

45. $y = e^x \ln x \quad y' = e^x \ln x + e^x\left(\frac{1}{x}\right) = e^x(\ln x + \frac{1}{x})$

46. $y = xe^x \quad y' = e^x + xe^x = e^x(1+x)$

47. $y = x^3e^x \quad y' = 3x^2e^x + x^3e^x = e^x(3x^2 + x^3)$

48. $y = x^2e^{-x} \quad y' = 2x\,e^{-x} - x^2e^{-x} = e^{-x}(2x - x^2)$

49. $g(t) = (e^{-t} + e^t)^3 \quad g'(t) = 3(e^{-t} + e^t)^2(-e^{-t} + e^t)$

50. $g(t) = e^{-3/t^2} \quad g'(t) = \left(e^{-\frac{3}{t^2}}\right)(-3)(-2)(t^{-1}) = \frac{6}{t}e^{-\frac{3}{t^2}}$

51. $y = \ln(1 + e^{2x}) \quad y' = \frac{1}{1+e^{2x}}(2e^{2x})$

52. $y = \ln(\frac{1+e^x}{1-e^x})$

$y = \ln(1 + e^x) - \ln(1 - e^x) \quad y' = \frac{e^x}{1+e^x} - \frac{-e^x}{1-e^x} = \frac{e^x}{1+e^x} + \frac{e^x}{1-e^x}$

53. $y = \frac{2}{e^x+e^{-x}} \quad y' = (-2)\frac{e^x-e^{-x}}{(e^x+e^{-x})^2} = 2\frac{e^{-x}-e^x}{(e^x+e^{-x})^2}$

54. $y = \frac{e^x-e^{-x}}{2} \quad y' = \frac{e^x+e^{-x}}{2}$

55. $y = \frac{e^x+1}{e^x-1} \quad y' = \frac{e^x(e^x-1)-(e^x+1)e^x}{(e^x-1)^2} = \frac{-2e^x}{(e^x-1)^2}$

56. $y = \frac{e^{2x}}{e^{2x}+1} \quad y = 1 - \frac{1}{e^{2x}+1} \quad y' = \frac{2e^{2x}}{(e^{2x}+1)^2}$

57. $y = e^x(\sin x + \cos x) \quad y' = e^x(\sin x + \cos x) + e^x(\cos x - \sin x) = 2e^x \cos x$

58. $y = \ln e^x \quad y = \ln e^x = x \quad y' = 1$

59. $F(x) = \int_\pi^{\ln x} \cos e^t\, dt$

$\frac{d}{dx}(\int_a^x f(t)\,dt) = f(x)$ The Fundamental Theorem of Calculus

$\frac{d}{dx}\left(\int_a^{u(x)} f(t)\,dt\right) = f(u(x)) \cdot u'$ Chain rule

$\frac{d}{dx}\left(\int_\pi^{\ln x} \cos e^t\,dt\right) = \cos e^{\ln x} \cdot (\ln x)' = \frac{\cos x}{x}$ $u(x) = \ln x$

60. $F(x) = \int_0^{e^{2x}} \ln(t+1)\,dt$

$\frac{d}{dx}\left(\int_a^{u(x)} f(t)\,dt\right) = f(u(x)) \cdot u'$ Chain rule

$\frac{d}{dx}\left(\int_0^{e^{2x}} \ln(t+1)\,dt\right) = \ln(e^{2x}+1)\,(e^{2x})' = 2\ln(e^{2x}+1)\,e^{2x}$ $u(x) = e^{2x}$

Exercise 6: (p359-69) use implicit differentiation to find dy/dx.

69. $xe^y - 10x + 3y = 0$

$e^y + xe^y\frac{dy}{dx} - 10 + 3\frac{dy}{dx} = 0 \Rightarrow 3\frac{dy}{dx} + xe^y\frac{dy}{dx} = 10 - e^y \Rightarrow \frac{dy}{dx} = \frac{10-e^y}{3+xe^y}$

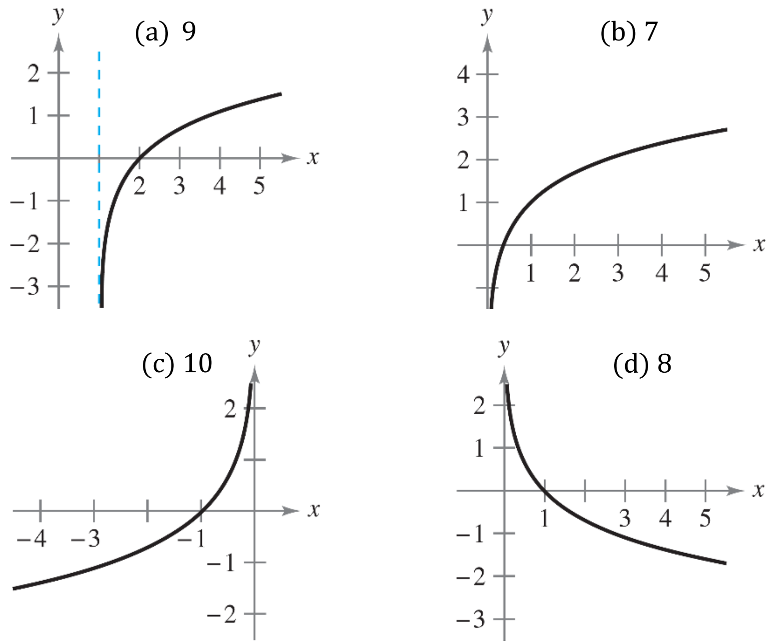
70. $e^{xy} + x^2 - y^2 = 10$

$e^{xy}\left(y + x\frac{dy}{dx}\right) + 2x - 2y\frac{dy}{dx} = 0 \Rightarrow 2y\frac{dy}{dx} - e^{xy}x\frac{dy}{dx} = e^{xy}y + 2x \Rightarrow \frac{dy}{dx} = \frac{e^{xy}y+2x}{2y-e^{xy}x}$

Exercise 7: (p331-7), match the function with its graph.

7. $f(x) = \ln x + 1$ 8. $f(x) = -\ln x$

9. $f(x) = \ln(x-1)$ 10. $f(x) = -\ln(-x)$



习题答案

Exercise 8: (p332-63), find the derivative of the function.

63. $y=\ln\sqrt{\frac{x+1}{x-1}}\quad y=\frac{1}{2}[\ln(x+1)-\ln(x-1)]$

$y'=\frac{1}{2}(\frac{1}{x+1}-\frac{1}{x-1})=\frac{-1}{(x+1)(x-1)}$

64. $y=\ln\sqrt[3]{\frac{x-1}{x+1}}\quad y=\frac{1}{3}[\ln(x-1)-\ln(x+1)]$

$y'=\frac{1}{3}\left(\frac{1}{x-1}-\frac{1}{x+1}\right)=\frac{2}{3(x-1)(x+1)}$

65. $y=\ln\frac{\sqrt{4+x^2}}{x}\quad y=\frac{1}{2}\ln(4+x^2)-\ln x\qquad y'=\frac{1}{2}\frac{2x}{4+x^2}-\frac{1}{x}=\frac{x}{4+x^2}-\frac{1}{x}$

66. $y=\ln(x+\sqrt{4+x^2})$

$y'=\frac{1}{x+\sqrt{4+x^2}}\left(1+\frac{1}{2}\frac{2x}{\sqrt{4+x^2}}\right)=\frac{1}{x+\sqrt{4+x^2}}\left(1+\frac{x}{\sqrt{4+x^2}}\right)=\frac{1}{x+\sqrt{4+x^2}}\left(\frac{\sqrt{4+x^2}+x}{\sqrt{4+x^2}}\right)=\frac{1}{\sqrt{4+x^2}}$

方法 2: $\quad Let\ u=x+\sqrt{4+x^2}, u'=1+\frac{1}{2}(4+x^2)^{-\frac{1}{2}}(2x)=1+\frac{x}{\sqrt{4+x^2}}$

$f'(x)=\frac{u'}{u}=\frac{1+\frac{x}{\sqrt{4+x^2}}}{x+\sqrt{4+x^2}}==\frac{(\sqrt{4+x^2}+x)/\sqrt{4+x^2}}{x+\sqrt{4+x^2}}=\frac{1}{\sqrt{4+x^2}}$

69. $y=\ln|\sin x|$

$y=\begin{cases}\ln(\sin x) & x\in[2k\pi,2k\pi+\pi)\\ \ln(-\sin x) & x\in[2k\pi-\pi,2k\pi)\end{cases}$

$[\ln(\sin x)]'=\frac{1}{\sin x}\cos x$

$[\ln(-\sin x)]'=\frac{1}{-\sin x}(-\cos x)=\frac{1}{\sin x}\cos x$

$y'=\cot x$

方法 2: $\quad y'=\frac{u'}{u}=\frac{\cos x}{sin\ x}=\cot x$

70. $y=\ln|\csc x|\qquad y=\ln|1/\sin x|$

$y=\begin{cases}\ln(1/\sin x)=\ln 1-\ln(\sin x) & x\in[2k\pi,2k\pi+\pi)\\ \ln[1/(-\sin x)]=\ln 1-\ln(-\sin x) & x\in[2k\pi-\pi,2k\pi)\end{cases}$

$[\ln 1-\ln(\sin x)]'=\frac{-1}{\sin x}\cos x$

$[\ln 1-\ln(-\sin x)]'=\frac{-1}{-\sin x}(-\cos x)=\frac{-1}{\sin x}\cos x$

$y'=-\cot x$

75. $f(x)=\int_2^{\ln(2x)}(t+1)dt$

$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right)=f(x)$ The Fundamental Theorem of Calculus

$\frac{d}{dx}\left(\int_a^{u(x)} f(t)\,dt\right)=f\big(u(x)\big)\cdot u'$ Chain rule

$\frac{d}{dx}\left(\int_2^{\ln(2x)}(t+1)dt\right)=(\ln 2x+1)\cdot(\ln 2x)'=(\ln 2x+1)\frac{2}{2x}=\frac{\ln 2x+1}{x}$

$u(x)=ln\ 2x$

76. $g(x)=\int_1^{\ln x}(t^2+3)dt$

$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right)=f(x)$ The Fundamental Theorem of Calculus

$\frac{d}{dx}\left(\int_a^{u(x)} f(t)\,dt\right)=f\big(u(x)\big)\cdot u'$ Chain rule

$\frac{d}{dx}\left(\int_1^{\ln x}(t^2+3)dt\right)=[(\ln x)^2+3]\cdot(\ln x)'=\frac{(\ln x)^2+3}{x}$ $\quad u(x)=ln\ x$

Exercise 9: (p332-77) (a) find an equation of the tangent line to the graph of f at the given point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the derivative feature of a graphing utility to confirm your results.

77. $f(x)=3x^2-\ln x,\quad(1,3)$

$f'(x)=6x-\frac{1}{x}$

The slop of the tangent at $(1,3)$ is $f'(1)=6-1=5$

Point-slop equation: $y-y_0=k(x-x_0)\Rightarrow y-3=5(x-1)\Rightarrow y=5x-2$

78. $f(x) = 4 - x^2 - \ln\left(\frac{1}{2}x + 1\right), \quad (0,4)$

$f'(x) = 0 - 2x - \left(\frac{1}{2}x + 1\right)^{-1}\left(\frac{1}{2}\right) = -2x - \frac{1}{x+2}$

The slop of the tangent at $(0,4)$ is $f'(0) = 0 - \frac{1}{2} = \frac{-1}{2}$

Point-slop equation: $y - y_0 = k(x - x_0) \Rightarrow y - 4 = \frac{-1}{2}(x - 0) \Rightarrow y = \frac{-1}{2}x + 4$

Exercise 10: (p332-101), use logarithmic differentiation to find dy/dx.

101. $y = x\sqrt{x^2+1}, x > 0$

$y = x\sqrt{x^2+1} \Rightarrow \ln y = \ln x + \frac{1}{2}\ln(x^2+1)$

$\Rightarrow \frac{d}{dx}(\ln y) = \frac{1}{x} + \frac{1}{2}\frac{2x}{(x^2+1)} = \frac{1}{x} + \frac{x}{x^2+1} = \frac{x^2+1+x^2}{x(x^2+2)} = \frac{2x^2+1}{x(x^2+1)}$

$\frac{d}{dx}(\ln y) = \frac{1}{y}\frac{dy}{dx} \Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = y\frac{d}{dx}(\ln y) = x\sqrt{x^2+1}\frac{2x^2+1}{x(x^2+1)} = \frac{2x^2+1}{\sqrt{x^2+1}}$

102. $y = \sqrt{x^2(x+1)(x+2)}, x > 0$

$y = \sqrt{x^2(x+1)(x+2)} \Rightarrow \ln y = \frac{1}{2}[2\ln x + \ln(x+1) + \ln(x+2)]$

$\Rightarrow \frac{d}{dx}(\ln y) = \frac{1}{2}\left(\frac{2}{x} + \frac{1}{x+1} + \frac{1}{x+2}\right) = \frac{4x^2+9x+4}{2x(x+1)(x+2)}$

$\frac{d}{dx}(\ln y) = \frac{1}{y}\frac{dy}{dx} \Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = y\frac{d}{dx}(\ln y)$

$= \sqrt{x^2(x+1)(x+2)}\left(\frac{4x^2+9x+4}{2x(x+1)(x+2)}\right) = \frac{4x^2+9x+4}{2\sqrt{(x+1)(x+2)}}$

103. $y = \frac{x^2\sqrt{3x-2}}{(x+1)^2}, x > \frac{2}{3}$

$y = \frac{x^2\sqrt{3x-2}}{(x+1)^2} \Rightarrow \ln y = 2\ln x + \frac{1}{2}\ln(3x-2) - 2\ln(x+1)$

$\Rightarrow \frac{\mathrm{d}}{\mathrm{dx}}(\ln y) = \frac{2}{x} + \frac{1}{2}\left(\frac{3}{3x-2}\right) - 2\left(\frac{1}{x+1}\right)$

$\frac{d}{dx}(\ln y) = \frac{1}{y}\frac{dy}{dx} \Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = y\frac{d}{dx}(\ln y) = \frac{x^2\sqrt{3x-2}}{(x+1)^2}\left[\frac{2}{x} + \frac{1}{2}\left(\frac{3}{3x-2}\right) - 2\left(\frac{1}{x+1}\right)\right] = \frac{2x\sqrt{3x-2}}{(x+1)^2} +$

$+ \frac{3x^2}{2(x+1)^2\sqrt{3x-2}} - \frac{2x^2\sqrt{3x-2}}{(x+1)^3}$

或者 $\frac{x^2\sqrt{3x-2}}{(x+1)^2}\left[\frac{2}{x} + \frac{1}{2}\left(\frac{3}{3x-2}\right) - 2\left(\frac{1}{x+1}\right)\right] = \frac{x^2\sqrt{3x-2}}{(x+1)^2}\left(\frac{3x^2+15x-8}{2x(3x-2)(x+1)}\right) = \frac{3x^2+15x-8}{2(x+1)^3\sqrt{3x-2}}$

104. $y = \sqrt{\frac{x^2-1}{x^2+1}}, x > 1$

$y = \sqrt{\frac{x^2-1}{x^2+1}} \Rightarrow \ln y = \frac{1}{2}(\ln(x^2-1) - \ln(x^2+1))$

$\Rightarrow \frac{\mathrm{d}}{\mathrm{dx}}(\ln y) = \frac{1}{2}\left(\frac{2x}{x^2-1} - \frac{2x}{x^2+1}\right) = \frac{x}{x^2-1} - \frac{x}{x^2+1}$

$\frac{d}{dx}(\ln y) = \frac{1}{y}\frac{dy}{dx} \Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = y\frac{d}{dx}(\ln y) = \sqrt{\frac{x^2-1}{x^2+1}}\left(\frac{x}{x^2-1} - \frac{x}{x^2+1}\right) = \frac{x}{\sqrt{x^2+1}\sqrt{x^2-1}} -$

$\frac{x\sqrt{x^2-1}}{(x^2+1)^{\frac{3}{2}}} = \frac{2x}{(x^2+1)^{\frac{3}{2}}\sqrt{x^2-1}}$

105. $y = \frac{x(x-1)^{3/2}}{\sqrt{x+1}}, \quad x > 1$

$y = \frac{x(x-1)^{3/2}}{\sqrt{x+1}} \Rightarrow \ln y = \ln x + \frac{3}{2}\ln(x-1) - \frac{1}{2}\ln(x+1)$

$\Rightarrow \frac{\mathrm{d}}{\mathrm{dx}}(\ln y) = \frac{1}{x} + \frac{3}{2}\left(\frac{1}{x-1}\right) - \frac{1}{2}\left(\frac{1}{x+1}\right) = \frac{2x^2+2x-1}{x^3-x}$

$\frac{d}{dx}(\ln y) = \frac{1}{y}\frac{dy}{dx} \Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = y\frac{d}{dx}(\ln y) = \frac{x(x-1)^{\frac{3}{2}}}{\sqrt{x+1}}\left[\frac{1}{x} + \frac{3}{2(x-1)} - \frac{1}{2(x+1)}\right]$

$= \frac{x(x-1)^{3/2}}{\sqrt{x+1}}\left[\frac{2x^2+2x-1}{x^3-x}\right]$

106. $y = \frac{(x+1)(x-2)}{(x-1)(x+2)}, \quad x > 2$

$y = \frac{(x+1)(x-2)}{(x-1)(x+2)} \Rightarrow \ln y = \ln(x+1) + \ln(x-2) - \ln(x-1) - \ln(x+2)$

$\Rightarrow \frac{\mathrm{d}}{\mathrm{dx}}(\ln y) = \frac{1}{x+1} + \frac{1}{x-2} - \frac{1}{x-1} - \frac{1}{x+2} = \frac{2x^2+4}{x^4-5x^2+4}$

$\frac{d}{dx}(\ln y) = \frac{1}{y}\frac{dy}{dx} \Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = y\frac{d}{dx}(\ln y) = \frac{(x+1)(x-2)}{(x-1)(x+2)}\left(\frac{1}{x+1} + \frac{1}{x-2} - \frac{1}{x-1} - \frac{1}{x+2}\right)$

Exercise 1: Known $2^a=5^b=m$, and $\frac{1}{a}+\frac{1}{b}=2$, Find m.

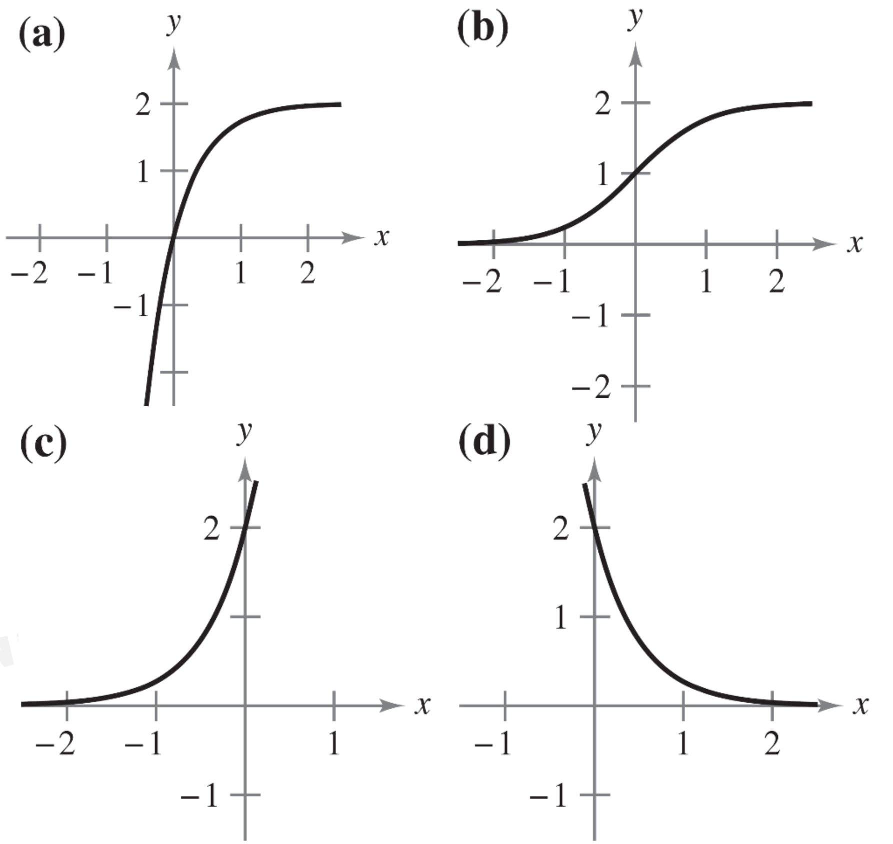
Exercise 2: 已知 $\log_{14}7=a$, $\log_{14}5=b$, 用 a,b 表示 $\log_{35}28$.

Exercise 3: (p358-24) Use a graphing utility to graph the function. Use the graph to determine any asymptotes of the function.

(a) $f(x)=\frac{8}{1+e^{-0.5x}}$

(b) $g(x)=\frac{8}{1+e^{-0.5/x}}$

Exercise 4: (p358-25), match the equation (labeled a, b, c and d) with the correct graph. Assume that a and C are positive real numbers.



25 $y=Ce^{ax}$

26 $y=Ce^{-ax}$

27 $y=C(1-e^{-ax})$

28 $y=\frac{C}{1+e^{-ax}}$

Exercise 5: (p359-45), find the derivative

45. $y = e^x \, ln \, x$

46. $y = xe^x$

47. $y = x^3 e^x$

48. $y = x^2 e^{-x}$

49. $g(t) = (e^{-t} + e^t)^3$

50. $g(t) = e^{-3/t^2}$

51. $y = \ln(1 + e^{2x})$

52. $y = \ln(\frac{1+e^x}{1-e^x})$

53. $y = \frac{2}{e^x + e^{-x}}$

54. $y = \frac{e^x - e^{-x}}{2}$

55. $y = \frac{e^x + 1}{e^x - 1}$

56. $y = \frac{e^{2x}}{e^{2x} + 1}$

57. $y = e^x(\sin x + \cos x)$

58. $y = \ln e^x$

59. $F(x) = \int_{\pi}^{\ln x} \cos e^t \, dt$

60. $F(x) = \int_0^{e^{2x}} \ln(t+1) \, dt$

Exercise 6: (p359-69) use implicit differentiation to find dy/dx.

69. $xe^y - 10x + 3y = 0$

70. $e^{xy} + x^2 - y^2 = 10$

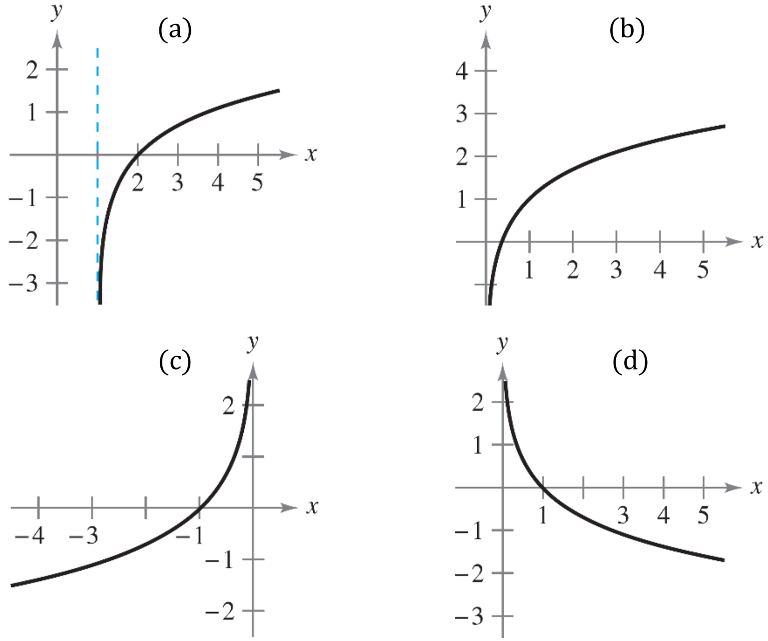
Exercise 7: (p331-7), match the function with its graph.

7. $f(x) = \ln x + 1$

8. $f(x) = -\ln x$

9. $f(x) = \ln(x - 1)$

10. $f(x) = -\ln(-x)$



Exercise 8: (p332-63), find the derivative of the function.

63. $y = \ln \sqrt{\frac{x+1}{x-1}}$

64. $y = \ln \sqrt[3]{\frac{x-1}{x+1}}$

65. $y = \ln \frac{\sqrt{4+x^2}}{x}$

66. $y = \ln(x + \sqrt{4 + x^2})$

69. $y = \ln|\sin x|$

70. $y = \ln|\csc x|$

75. $f(x) = \int_2^{\ln(2x)} (t+1)dt$

76. $g(x) = \int_1^{\ln x} (t^2+3)dt$

Exercise 9: (p332-77) (a) find an equation of the tangent line to the graph of f at the given point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the derivative feature of a graphing utility to confirm your results.

77. $f(x) = 3x^2 - \ln x$, $(1,3)$

78. $f(x) = 4 - x^2 - \ln\left(\frac{1}{2}x + 1\right), \quad (0,4)$

Exercise 10: (p332-101), use logarithmic differentiation to find dy/dx.

101. $y = x\sqrt{x^2+1}, x > 0$

102. $y = \sqrt{x^2(x+1)(x+2)}, x > 0$

103. $y = \frac{x^2\sqrt{3x-2}}{(x+1)^2}, x > \frac{2}{3}$

104. $y = \sqrt{\frac{x^2-1}{x^2+1}}, x > 1$

105. $y = \frac{x(x-1)^{3/2}}{\sqrt{x+1}}, x > 1$

106. $y = \frac{(x+1)(x-2)}{(x-1)(x+2)}, x > 2$